# Mthe 237 Final Exam

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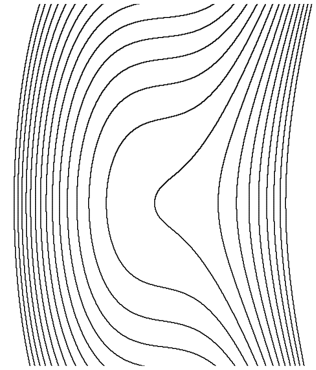

**Instructions:** This exam is three hours in duration.

Please write your answers in the booklets provided. Hand in both the booklets and the question paper.

To receive full credit, you must justify your answers. Answers with little or no justification will receive little or no credit.

Calculators, data sheets, notes, and other aids are not permitted. A table of Laplace transforms is provided on the last two pages.

**Good Luck!**

**Please Note:** Proctors are unable to respond to queries about the interpretation of exam questions. Do your best to answer exam questions as written.

Student Number: ____________________

| 1 | 2 | 3 | 4 | 5 | Total |
|---|---|---|---|---|---|
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# Table of Laplace Transforms

Notation: $\mathbb{N} = \{0, 1, 2, \ldots\}$ natural numbers, $\mathbb{R}$ real numbers.

$$u_a(t) = u_0(t-a) = \begin{cases} 1 & t \geq a \\ 0 & \text{otherwise} \end{cases}, \quad \text{unit step function with jump at } t = a.$$

## 1 General properties

| $f(t),\ t \geq 0$ | $\mathscr{L}[f](s) = \int_0^\infty f(t) e^{-st}\,dt$ |
|---|---|
| $(f+g)(t)$ | $\mathscr{L}[f](s) + \mathscr{L}[g](s)$ |
| $(cf)(t),\ c \in \mathbb{R}$ | $c\mathscr{L}[f](s)$ |
| $\frac{df}{dt}(t)$ | $s\mathscr{L}[f](s) - f(0)$ |
| $\frac{d^2 f}{dt^2}(t)$ | $s^2\mathscr{L}[f](s) - sf(0) - \frac{df}{dt}(0)$ |
| $\frac{d^r f}{dt^r}(t),\ r \in \mathbb{N}$ | $s^r\mathscr{L}[f](s) - s^{r-1}f(0) - s^{r-2}\frac{df}{dt}(0) - \cdots - \frac{d^{r-1}f}{dt^{r-1}}(0)$ |
| $u_a(t)f(t-a),\ a \geq 0,$ | $e^{-as}\mathscr{L}[f](s)$ |
| $e^{\sigma s} f(t),\ \sigma \in \mathbb{R}$ | $\mathscr{L}[f](s-\sigma)$, whereever converges |
| $(f * g)(t) = \int_0^t f(u)g(t-u)\,du$ | $\mathscr{L}[f](s)\,\mathscr{L}[g](s)$ |

## 2 Transforms of Quasipolynomials and the Unit Step Functions

| | |
|---|---|
| $1$ | $\frac{1}{s}, \qquad s > 0$ |
| $t$ | $\frac{1}{s^2}, \qquad s > 0$ |
| $t^k,\ k \in \mathbb{N}$ | $\frac{k!}{s^{k+1}}, \qquad s > 0$ |
| $e^{\sigma t},\ \sigma \in \mathbb{R}$ | $\frac{1}{s-\sigma}, \qquad s > \sigma$ |
| $t\, e^{\sigma t},\ \sigma \in \mathbb{R}$ | $\frac{1}{(s-\sigma)^2}, \qquad s > \sigma$ |
| $t^k\, e^{\sigma t},\ \sigma \in \mathbb{R}, k \in \mathbb{N}$ | $\frac{k!}{(s-\sigma)^{k+1}}, \qquad s > \sigma$ |
| $\cos(\omega t),\ \omega \in \mathbb{R}$ | $\frac{s}{s^2+\omega^2}, \qquad s > 0$ |
| $\sin(\omega t),\ \omega \in \mathbb{R}$ | $\frac{\omega}{s^2+\omega^2}, \qquad s > 0$ |
| $t\,\cos(\omega t),\ \omega \in \mathbb{R}$ | $\frac{s^2-\omega^2}{(s^2+\omega^2)^2}, \qquad s > 0$ |
| $t\,\sin(\omega t),\ \omega \in \mathbb{R}$ | $\frac{2\omega s}{(s^2+\omega^2)^2}, \qquad s > 0$ |
| $t^k\,\cos(\omega t),\ \omega \in \mathbb{R}, k \in \mathbb{N}$ | $(-1)^k \frac{d^k}{ds^k}\left(\frac{s}{s^2+\omega^2}\right), \qquad s > 0$ |
| $t^k\,\sin(\omega t),\ \omega \in \mathbb{R}, k \in \mathbb{N}$ | $(-1)^k \frac{d^k}{ds^k}\left(\frac{\omega}{s^2+\omega^2}\right), \qquad s > 0$ |
| $e^{\sigma t}\cos(\omega t),\ \omega \in \mathbb{R}, \sigma \in \mathbb{R}$ | $\frac{s-\sigma}{(s-\sigma)^2+\omega^2}, \qquad s > \sigma$ |
| $e^{\sigma t}\sin(\omega t),\ \omega \in \mathbb{R}, \sigma \in \mathbb{R}$ | $\frac{\omega}{(s-\sigma)^2+\omega^2}, \qquad s > \sigma$ |
| $t\,e^{\sigma t}\cos(\omega t),\ \omega \in \mathbb{R}, \sigma \in \mathbb{R}$ | $\frac{(s-\sigma)^2-\omega^2}{((s-\sigma)^2+\omega^2)^2}, \qquad s > \sigma$ |
| $t\,e^{\sigma t}\sin(\omega t),\ \omega \in \mathbb{R}, \sigma \in \mathbb{R}$ | $\frac{2(s-\sigma)\omega}{((s-\sigma)^2+\omega^2)^2}, \qquad s > \sigma$ |
| $u_a(t),\ a \geq 0$ | $\frac{e^{-as}}{s}, \qquad s > 0$ |