MTHE 227 Problem Set 2

Due Wednesday September 28 2016 at the beginning of class

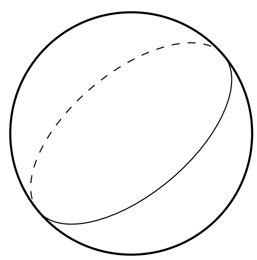
1 (Great Circles)**.** The intersection of a sphere with a plane passing through its center is called a *great circle*. Let Γ be the great circle that is the intersection of the plane $x+y+z=0$ with the sphere $x^2+y^2+z^2=R^2$ of radius R centered at the origin.

(a) Find a parametrization of Γ. (*Suggestion:* Begin by finding two perpendicular vectors lying in the plane $x+y+z=0$.)

(b) Check that the arclength of Γ is equal to $2\pi R$.

(c) If $t \mapsto (x(t), y(t), z(t))$ is a parametrization of Γ, explain why $t \mapsto (x(t), y(t), -z(t))$ is a parametrization of the great circle Γ' cut out from the same sphere by $x+y-z=0$.

Optional Problem. Find the points of intersection of Γ and Γ', and find the angle of intersection between the tangent lines to Γ and Γ' at these points.



2 (Velocity Perpendicular to Position)**.** For a pair of parametrized paths $\mathbf{q}(t)$, $\mathbf{r}(t)$ in $\mathbb{R}^3$, show that

$$\frac{d}{dt}\left(\mathbf{q}(t)\cdot\mathbf{r}(t)\right) = \mathbf{q}'(t)\cdot\mathbf{r}(t) + \mathbf{q}(t)\cdot\mathbf{r}'(t)$$

(here $\cdot$ denotes the dot product and $'$ the derivative with respect to t). Apply this identity to show the following: if the velocity of a parametrized path is always perpendicular to its position, then the curve traced out by the parametrization lies on the surface of a sphere.

3 (Flow Lines)**.** (a) Check that the path $t \mapsto (2\cos(2t), \sin 2t)$ is a flow line of the vector field $\mathbf{F}(x,y) = (-4y,\, x)$ on $\mathbb{R}^2$. Sketch the vector field, the path, and check that the path is everywhere tangent to $\mathbf{F}$.

(b) Find the flow lines of the vector field $\mathbf{G}(x,y) = (1,\, -y^2)$, defined on the first quadrant $\{(x,y) : x > 0,\ y > 0\}$ of $\mathbb{R}^2$. Which flow line passes through the point $(1,1)$? (*Hint:* It may help to find the derivative $\frac{d}{dt}(1/t)$.)