L1: Intro

September 12, 2016 1:30 PM

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JEFF 111

Help Centre OPEN!
(Jeff 201)

Office Hours: Mon 1:30 - 2:30?
(After Lecture)

__Grading:__
Homework : 30% (Best 10 / 12)
Midterm : 20% (time TBA → mid-late oct.)
Final : 50%

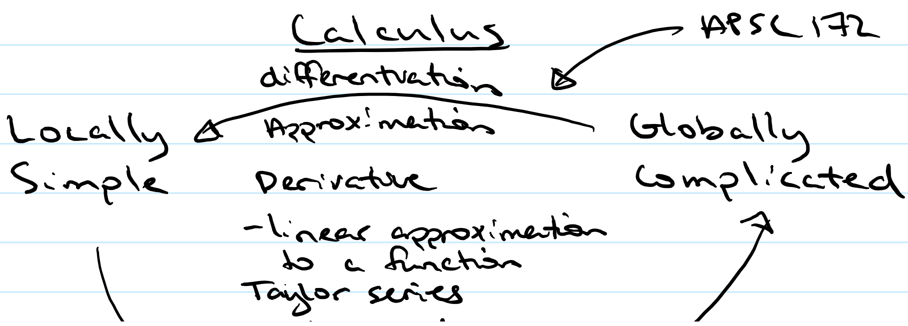
http://www.mast.queensu.ca/~smirnovi/mthe227

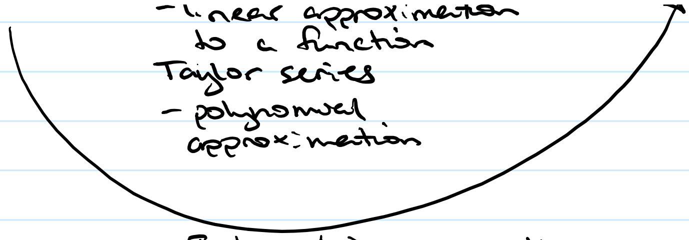
__Homework:__
- practice problems (not graded), a few per lecture.
- problem sets - graded 3-4 problems a week.
  - ↳ 1st is up, due next week.

__Textbook:__
- James Stewart
  Calculus 8th edition (2015? older works too)
- Courant - a bit dated, but very good.
  Integral & Differential Calculus, Vol 2.
- Spivak - Calculus on Manifolds
  - ↳ only for curious.

OnQ = new moodle.
page will exist ... eventually... (will only be for marks)

for homework → group work encouraged
↳ do individual writeups.





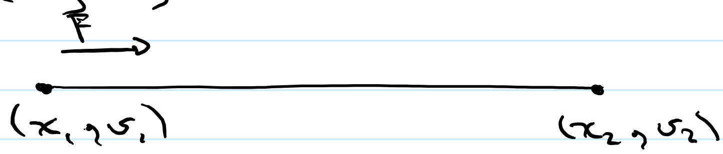
Integration ⟵ this course

Example 1: Work on a free particle.

Local Picture: The configuration of a free particle is determined by the pair (x, v)
↗ position ↖ velocity

The total energy of a particle is its kinetic energy $\frac{1}{2}mv^2$

Suppose the particle moves along a straight line, acted upon by a constant force $\vec{F}$ that is parallel to the path



Let t = time to go from x_1 to x_2

Q: What's the change in energy.

From Newton's second law, $\vec{F} = m\vec{a}$

$$v_2 - v_1 = t \cdot a = t\frac{F}{m}, \text{ or}$$

$$(*) \quad m(v_2 - v_1) = Ft \qquad (**) \text{ Also, } v_{avg} = \frac{v_2 + v_1}{2} = \frac{x_2 - x_1}{t}$$

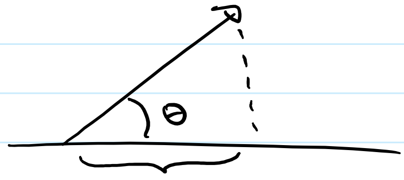
multiplying $(*)$ by $(**)$, we get

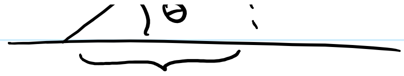
$$m(v_2 - v_1)\frac{v_2 + v_1}{2} = \underbrace{F(x_2 - x_1)}_{\text{work done by } F}$$

$$\frac{m}{2}(v_2^2 - v_1^2) =$$

If F is constant, but makes a constant angle θ with the path, the perpendicular component of F (to the path) doesn't affect its kinetic energy.

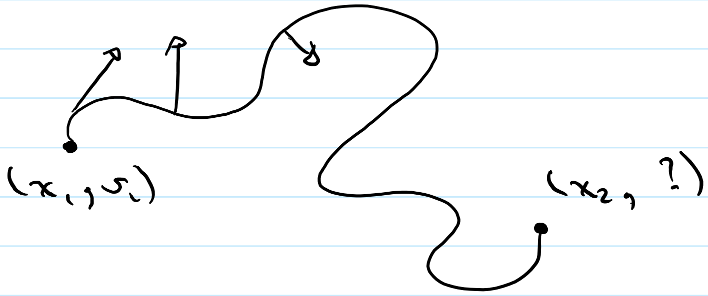
$$\frac{m}{2}(v_2^2 - v_1^2) = |\vec{F}|\cos\theta\,(x_2 - x_1)$$

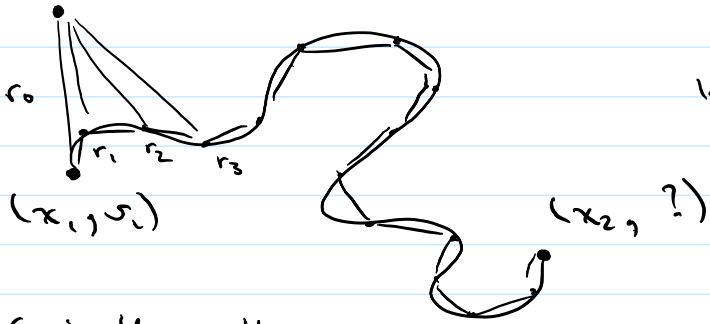




$\cos\theta = \vec{F}$ along the path

Q: what to do if the path is curved arbitrarily, and the force is allowed to vary continuously along the path?

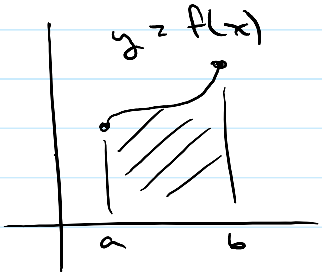




$$\lim_{|\Delta \vec{r}_i| \to 0} \sum_i F_i \cdot \Delta \vec{r}_i = W$$

$$\int_C \vec{F} \cdot d\vec{r}$$

*C is the path



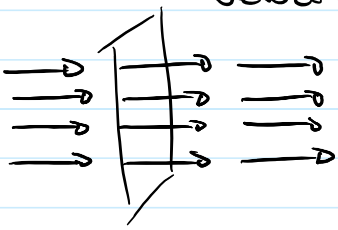
$\int_a^b f(x)\,dx$ is the area under the graph.

$\int$ is an elongated "S" and stands for sum (so does Σ in greek)

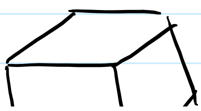
One way to define $\int_a^b f(x)\,dx$ is to partition $[a,b]$ into small segments (reimen sums)

2. Flux

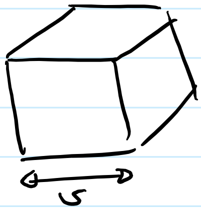
Local picture: Suppose we have a fluid flowing with uniform velocity everywhere in space.



Q: How much fluid flows through a rectangular region perpendicular to the flow?

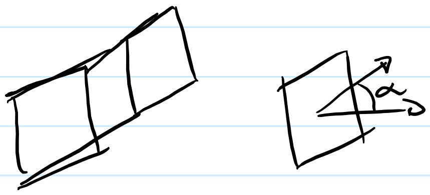


volume of fluid passing through in 1 sec is ... Area of the region

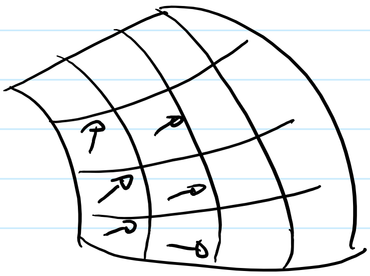


volume of fluid passing through in 1 sec
is v · Area of the region.

suppose the fluid makes an angle θ with the perpendicular vector to the region.



$v \cdot \cos\theta \cdot$ area of region.



Surface S

$\lim_{\Delta \vec{A} \to 0} \sum \vec{v}_i \cdot \Delta \vec{A}_i$ = total flow

$\iint_S \vec{v} \cdot \vec{n} \, dA$