MTHE 227 Practice Midterm

This is a practice midterm. It has the same format, and the same number and type of questions as the midterm on October 25. The practice midterm is not to be handed in.

Instructions (as they will appear on the midterm). There are six questions on this exam, totaling to 100 points. Attempt each question. The point worth of each question is specified below. The duration of the exam is two hours. Calculators, data sheets and other aids are not permitted. To receive full credit, you must explain your answers (except for the last question). Answers are to be recorded in an exam booklet handed out by the instructor.

Good luck!

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1 (10 points)**.** Let $f(x,y,z) = 3\sqrt{1+4x^2+4z}$. Let C be the segment of the space parabola traced out by $t \mapsto (t, t^2, t^2)$, $t \in [0,1]$. Compute $\int_C f\,ds$.

2. Let C be the ellipse $(x^2/4) + (y^2/9) = 1$ in $\mathbb{R}^2$.

(a) (7 points) Parametrize C.

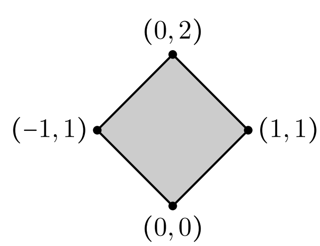
(b) (8 points) For each point of C, parametrize the line tangent to C at that point.

(If you are having trouble, you can instead parametrize the circle $x^2 + y^2 = R^2$, as well as its tangent lines, for a maximum of 10 points.)

3 (15 points)**.** Let C be the segment of the hyperbola $xy = 4$ from $(1,4)$ to $(4,1)$ in $\mathbb{R}^2$. Let $\mathbf{F}$ be the vector field $\mathbf{F}(x,y) = (x+y,\ x-y)$. Compute the work $\int_C \mathbf{F}\cdot\mathbf{dr}$ done by $\mathbf{F}$ along C.

4. (a) (6 points) Let R be the rectangle $[0,1]\times[0,1]$ in $\mathbb{R}^2$. Let $f(x,y) = y\cos(\pi xy)$. Compute $\iint_R f\,dA$.

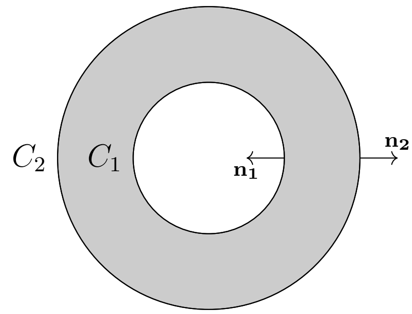
(b) (7 points) Let R be the following quadrilateral in $\mathbb{R}^2$:



Let $f(x,y) = xy + x + y + 1$. Set up $\iint_R f\,dA$ as an iterated integral, or a sum of iterated integrals. It is not necessary to evaluate the integral.

(c) (7 points) Compute $\displaystyle\int_0^2 \int_{y/2}^1 e^{-x^2}\,dxdy$. (Suggestion: Change the order of integration. The function e^{-x^2} has no elementary antiderivative.)

5. Let C_1 be the circle $x^2 + y^2 = 1$, with normal pointing toward the origin, and C_2 the circle $x^2 + y^2 = 4$, with normal pointing away from the origin.



Let $\mathbf{F}$ be the vector field

$$\mathbf{F}(x,y) = \left(xe^{x^2+y^2}, \; ye^{x^2+y^2} \right).$$

(a) (15 points) Parametrize the curves C_1 and C_2. Compute $\int_{C_1} \mathbf{F} \cdot \hat{\mathbf{n}}\, ds + \int_{C_2} \mathbf{F} \cdot \hat{\mathbf{n}}\, ds$.

(b) (5 points) Let $f(x,y)$ denote the function

$$\frac{\partial}{\partial x}\left(xe^{x^2+y^2}\right) + \frac{\partial}{\partial y}\left(ye^{x^2+y^2}\right).$$

Compute $f(x,y)$.

(c) (10 points) Let R be the region bounded by C_1 and C_2. Compute $\iint_R f\, dA$. (Suggestion: Use polar coordinates. You may need to integrate one of the terms by parts.)

Remark. As a check on your answers, you should find that $\int_{C_1} \mathbf{F} \cdot \hat{\mathbf{n}}\, ds + \int_{C_2} \mathbf{F} \cdot \hat{\mathbf{n}}\, ds = \iint_R f\, dA$ (Green's Theorem). However, you may not apply Green's Theorem in your solution of this question.

6 (10 points)**.** True or False? 2 points each. No justification necessary. No penalty for incorrect answers.

(a) The polar coordinates of every point of $\mathbb{R}^2$ are unique (for every point of $\mathbb{R}^2$, there is a unique pair (r,θ) such that (r,θ) are the polar coordinates of that point).

(b) For any conservative vector field, there exists a unique potential function.

(c) The value of the work done by a force represents the change in kinetic energy caused by that force between the two endpoints of the path.

(d) The flux of a vector field that is everywhere parallel to a path across that path is equal to zero.

(e) Any parametrized path has exactly two possible orientations.