

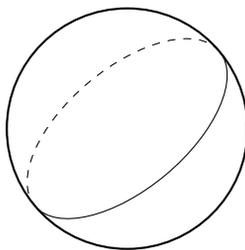
MTHE 227 PROBLEM SET 2

Due Wednesday September 28 2016 at the beginning of class

**1 (Great Circles).** The intersection of a sphere with a plane passing through its center is called a *great circle*. Let  $\Gamma$  be the great circle that is the intersection of the plane  $x + y + z = 0$  with the sphere  $x^2 + y^2 + z^2 = R^2$  of radius  $R$  centered at the origin.

- (a) Find a parametrization of  $\Gamma$ . (*Suggestion:* Begin by finding two perpendicular vectors lying in the plane  $x + y + z = 0$ .)
- (b) Check that the arclength of  $\Gamma$  is equal to  $2\pi R$ .
- (c) If  $t \mapsto (x(t), y(t), z(t))$  is a parametrization of  $\Gamma$ , explain why  $t \mapsto (x(t), y(t), -z(t))$  is a parametrization of the great circle  $\Gamma'$  cut out from the same sphere by  $x + y - z = 0$ .

*Optional Problem.* Find the points of intersection of  $\Gamma$  and  $\Gamma'$ , and find the angle of intersection between the tangent lines to  $\Gamma$  and  $\Gamma'$  at these points.



**2 (Velocity Perpendicular to Position).** For a pair of parametrized paths  $\mathbf{q}(t), \mathbf{r}(t)$  in  $\mathbb{R}^3$ , show that

$$\frac{d}{dt} (\mathbf{q}(t) \cdot \mathbf{r}(t)) = \mathbf{q}'(t) \cdot \mathbf{r}(t) + \mathbf{q}(t) \cdot \mathbf{r}'(t)$$

(here  $\cdot$  denotes the dot product and  $'$  the derivative with respect to  $t$ ). Apply this identity to show the following: if the velocity of a parametrized path is always perpendicular to its position, then the curve traced out by the parametrization lies on the surface of a sphere.

**3 (Flow Lines).** (a) Check that the path  $t \mapsto (2 \cos(2t), \sin 2t)$  is a flow line of the vector field  $\mathbf{F}(x, y) = (-4y, x)$  on  $\mathbb{R}^2$ . Sketch the vector field, the path, and check that the path is everywhere tangent to  $\mathbf{F}$ .

- (b) Find the flow lines of the vector field  $\mathbf{G}(x, y) = (1, -y^2)$ , defined on the first quadrant  $\{(x, y) : x > 0, y > 0\}$  of  $\mathbb{R}^2$ . Which flow line passes through the point  $(1, 1)$ ? (*Hint:* It may help to find the derivative  $\frac{d}{dt} (1/t)$ .)